# Vaje MAT VSP, 6. 1. 2021

4. Z uporabo Gaussove eliminacije poišči vse rešitve naslednjih sistemov enačb:

$$\text{(a)}\ \begin{aligned} x + y + 2z &= 3\\ 2x - y + 4z &= 0\\ 3x - y + z &= 1 \end{aligned} \qquad \text{(b)}\ \begin{aligned} 2x + y - z &= 0\\ x + z &= 5\\ x + y - 2z &= -5 \end{aligned} \qquad \text{(c)}\ \begin{aligned} 2y + z &= 5\\ x - y + 2z &= 2\\ x + y + 3z &= 1 \end{aligned}$$

b)

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 2 & 1 & -1 & 0\\ 1 & 0 & 1 & 5\\ 1 & 1 & -2 & -5 \end{array}\right] \end{array} \overset{(-1)}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5\\ 2 & 1 & -1 & 0\\ 1 & 1 & -2 & -5 \end{array}\right] \ /\cdot(-2) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5\\ 0 & 1 & -3 & -10\\ 0 & 1 & -3 & -10 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5\\ 0 & 1 & -3 & -10\\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Sistem ima neskončno rešitev.

Število parametrov = število neznank − število neničelnih vrstic v razširjeni matriki sistema po Gaussovi eliminaciji

Število parametrov $= 3 - 2 = \underline{\underline{1}}$

$$y - 3z = -10$$
$$y = 3z - 10$$

$$x + z = 5$$
$$x = -z + 5$$

Rešitve sistema:

$$x = -t + 5$$
$$y = 3t - 10 \qquad t \in \mathbb{R}$$
$$z = t$$

$$\text{(c)}\quad \begin{aligned} 2y + z &= 5\\ x - y + 2z &= 2\\ x + y + 3z &= 1 \end{aligned}$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \phantom{5} \end{array}\\ \left[\begin{array}{ccc|c} 0 & 2 & 1 & 5\\ 1 & -1 & 2 & 2\\ 1 & 1 & 3 & 1 \end{array}\right] \end{array} \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2\\ \underline{0} & 2 & 1 & 5\\ \underline{1} & 1 & 3 & 1 \end{array}\right] \sim -$$

$$\oplus\left[\begin{array}{ccc|c} 1 & -1 & 2 & 2\\ 0 & 2 & 1 & 5\\ 0 & \underline{-2} & -1 & 1 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} x & y & z & \phantom{5} \end{array}\\ \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2\\ 0 & 2 & 1 & 5\\ 0 & 0 & 0 & 6 \end{array}\right] \end{array}$$

$$0x + 0y + 0z = 6$$
$$0 = 6 \,//$$

Sistem nima rešitev.

5. Poišči predpis za kvadratno funkcijo $f(x) = ax^2 + bx + c$, katere graf gre skozi točke $A(-1,6)$, $B(1,0)$ in $C(2,3)$.

$$\overset{y}{f(x)} = ax^2 + bx + c$$

$$A(\overset{x}{-1}, \overset{y}{6}) : \; 6 = a(-1)^2 + b(-1) + c$$

$$\boxed{a - b + c = 6}$$

$$B(\overset{x}{1}, \overset{y}{0}) : \; 0 = a \cdot 1^2 + b \cdot 1 + c$$

$$\boxed{a + b + c = 0}$$

$C(\overset{x}{2}, \overset{y}{3}): \quad 3 = a \cdot 2^2 + b \cdot 2 + c$

$$\boxed{4a + 2b + c = 3}$$

Dobili smo sistem enačb:

$$\begin{aligned} a - b + c &= 6 \\ a + b + c &= 0 \\ 4a + 2b + c &= 3 \end{aligned}$$

$$\begin{array}{ccc} a & b & c \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & 1 & 0 \\ 4 & 2 & 1 & 3 \end{array}\right] \begin{array}{l} /\cdot(-4) \\ \sim \\ + \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & -2 & 0 & 6 \\ 0 & 6 & -3 & -21 \end{array}\right] \begin{array}{l} \\ /:2 \sim \\ /:3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & -1 & 0 & 3 \\ 0 & 2 & -1 & -7 \end{array}\right] \begin{array}{l} \\ 2 \sim \\ + \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & -1 \end{array}\right] \end{array}$$

$-c = -1 \quad /\cdot(-1)$

$$\boxed{c = 1}$$

$-b = 3 \quad /\cdot(-1)$

$$\boxed{b = -3}$$

$a - b + c = 6$

$$\begin{aligned} a &= 6 + b - c \\ a &= 6 - 3 - 1 \end{aligned}$$

$$\boxed{a = 2}$$

Kvadratna funkcija: $f(x) = 2x^2 - 3x + 1$.

6. Določi polmer in središče krožnice, ki gre skozi točke $A(-1,1)$, $B(0,2)$ in $C(6,-6)$.

Enačba krožnice: $(x-p)^2+(y-q)^2=r^2$

$S(p,q)$, polmer: $r$

Enačba krožnice v splošni obliki:

$$x^2+y^2+ax+by+c=0$$

$A(\overset{x}{-1},\overset{y}{1})$: $(-1)^2+1^2+a(-1)+b\cdot 1+c=0$

$$\boxed{-a+b+c=-2}$$

$B(\overset{x}{0},\overset{y}{2})$: $0^2+2^2+a\cdot 0+b\cdot 2+c=0$

$$\boxed{2b+c=-4}$$

$C(\overset{x}{6},\overset{y}{-6})$: $6^2+(-6)^2+a\cdot 6+b(-6)+c=0$

$$\boxed{6a-6b+c=-72}$$

Sistem enačb:

$$-a+b+c=-2$$
$$2b+c=-4$$
$$6a-6b+c=-72$$

$$
\begin{array}{c}
\\ 6\cdot \\ \oplus \\ \\
\end{array}
\begin{array}{c}
\begin{array}{cccc} a & b & c & \phantom{-72} \end{array}\\
\left[\begin{array}{ccc|c}
-1 & 1 & 1 & -2 \\
0 & 2 & 1 & -4 \\
6 & -6 & 1 & -72
\end{array}\right]
\end{array}
\sim
\begin{array}{c}
\begin{array}{cccc} a & b & c & \phantom{-84} \end{array}\\
\left[\begin{array}{ccc|c}
-1 & 1 & 1 & -2 \\
0 & 2 & 1 & -4 \\
0 & 0 & 7 & -84
\end{array}\right]
\end{array}
\uparrow
$$

$$7c = -84 \quad /:7$$

$$\boxed{c = -12}$$

$$2b + c = -4$$

$$2b = -4 - c$$
$$2b = -4 - (-12)$$
$$2b = 8 \quad /:2$$
$$\boxed{b = 4}$$

$$-a + b + c = -2$$
$$a = b + c + 2$$
$$a = 4 - 12 + 2$$
$$\boxed{a = -6}$$

Enačba krožnice skozi točke A, B in C:

$$x^2 + y^2 + ax + by + c = 0$$

$$\boxed{x^2 + y^2 - 6x + 4y - 12 = 0}$$

Dopolnimo do popolnega kvadrata:

$$\underbrace{x^2 \overset{/:2}{-6}x} + \underbrace{y^2 \overset{/:2}{+4}y} - 12 = 0$$

$$(x-3)^2 \underbrace{-9}_{-3^2} + (y+2)^2 \underbrace{-4}_{-2^2} - 12 = 0$$

$$(x-3)^2+(y+2)^2-25=0$$

$$(x-3)^2+(y\overset{-(-2)}{+2})^2=\overset{5^2}{25}$$

$$\left[(x-p)^2+(y-q)^2=r^2\right]$$

$$S(3,-2)\quad,\quad r=5$$

7. Z uporabo Gaussove eliminacije poišči vse rešitve naslednjih sistemov linearnih enačb:

(a)
$$\begin{aligned} 2x_1+2x_2-x_3+x_4&=4\\ 4x_1+3x_2-x_3+2x_4&=6\\ 8x_1+5x_2-3x_3+4x_4&=12\\ 3x_1+3x_2-2x_3+2x_4&=6 \end{aligned}$$

(b)
$$\begin{aligned} 2x_1+7x_2+3x_3+x_4&=5\\ x_1+3x_2+5x_3-2x_4&=3\\ x_1+5x_2-9x_3+8x_4&=1\\ 5x_1+18x_2+4x_3+5x_4&=12 \end{aligned}$$

(c)
$$\begin{aligned} 4x_1-3x_2+2x_3-x_4&=8\\ 3x_1-2x_2+x_3-3x_4&=7\\ 2x_1-x_2-5x_4&=6\\ 5x_1-3x_2+x_3-8x_4&=1 \end{aligned}$$

(d)
$$\begin{aligned} 3x_1+4x_2+x_3+2x_4&=3\\ 6x_1+8x_2+2x_3+5x_4&=7\\ 9x_1+12x_2+3x_3+10x_4&=13 \end{aligned}$$

d)
$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}\\ \left[\begin{array}{cccc|c} 3&4&1&2&3\\ \underline{6}&8&2&5&7\\ \underline{9}&12&3&10&13 \end{array}\right] \end{array} /(-3) \sim \left[\begin{array}{cccc|c} 3&4&1&2&3\\ 0&0&0&1&1\\ 0&0&0&4&4 \end{array}\right] /:4 \sim$$

$$\sim \left[\begin{array}{cccc|c} 3&4&1&2&3\\ 0&0&0&1&1\\ \cancel{0}&\cancel{0}&\cancel{0}&\cancel{1}&\cancel{1} \end{array}\right] - \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}\\ \left[\begin{array}{cccc|c} 3&4&1&2&3\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{array}\right] \end{array}$$

Imamo neskončno rešitev.

Število parametrov $= 4-2=2$

$x_4 = 1$

$3x_1 + 4x_2 + x_3 + 2x_4 = 3$

$x_3 = 3 - 3x_1 - 4x_2 - 2x_4$ ← 1

$x_3 = 1 - 3x_1 - 4x_2$

Družina rešitev:

$x_1 = t$

$x_2 = r \qquad t, r \in \mathbb{R}$

$x_3 = 1 - 3t - 4r$

$x_4 = 1$

(c)
$$
\begin{aligned}
4x_1 - 3x_2 + 2x_3 - x_4 &= 8\\
3x_1 - 2x_2 + x_3 - 3x_4 &= 7\\
2x_1 - x_2 \phantom{{}+ x_3} - 5x_4 &= 6\\
5x_1 - 3x_2 + x_3 - 8x_4 &= 1
\end{aligned}
$$

$$
\begin{array}{c c c}
 & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} & \\
(-3) & \left[\begin{array}{cccc|c} 4 & -3 & 2 & -1 & 8 \\ 3 & -2 & 1 & -3 & 7 \\ 2 & -1 & 0 & -5 & 6 \\ 5 & -3 & 1 & -8 & 1 \end{array}\right] & \begin{array}{l} /\cdot 5 \\ + \\ /(-2) \\ /(-4) \end{array} \\
4 & & 
\end{array}
\sim
$$

$$
\left[\begin{array}{cccc|c} 4 & -3 & 2 & -1 & 8 \\ 0 & 1 & -2 & -9 & 4 \\ 0 & -1 & 2 & 9 & -4 \\ 0 & -3 & 6 & 27 & 36 \end{array}\right] \begin{array}{l} \\ \\ \\ /:3 \end{array} \sim
$$

$$\left[\begin{array}{cccc|c} 4 & -3 & 2 & -1 & 8 \\ 0 & 1 & -2 & -9 & 4 \\ 0 & \underline{-1} & 2 & 9 & -4 \\ 0 & \underline{-1} & 2 & 9 & 12 \end{array}\right] \sim \left[\begin{array}{cccc|c} 4 & -3 & 2 & -1 & 8 \\ 0 & 1 & -2 & -9 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 \end{array}\right] //$$

Sistem nima rešitev.