

Discrete structures VSP: computational preexam

January 11th, 2021

Time: 60 minutes. You are allowed to use 2 A4-sized sheets with formulae. Electronic devices are not allowed. Justify your answers!

Solve each task onto a separate page. Each page should be signed in the upper right corner, the task number should be clearly marked. Scan the tasks in increasing order. Thank you!

1. [30 marks] Students Alice, Bill, Celia and Dan made the following statements:

Alice: "If Celia copied, then Bill copied, too."

Bill: "Celia copied or Dan did not copy."

Celia: "Bill definitely did not copy."

Dan: "Alice or me copied. (Maybe both.)"

(a) Use basic statements:

$a \equiv$ Alice copied,

$c \equiv$ Celia copied,

$b \equiv$ Bill copied,

$d \equiv$ Dan copied,

to express student statements above in the language of propositional calculus.

(b) Write down two logical inferences with statements of the part (a) as premises, one with conclusion a , other with conclusion $a \Rightarrow c$.

(c) Determine whether inferences obtained in part (b) are valid. (Valid inference should be proved formally, invalid inference should have a counterexample.)

2. [35 marks] Relation R on the set \mathbb{N} is defined as

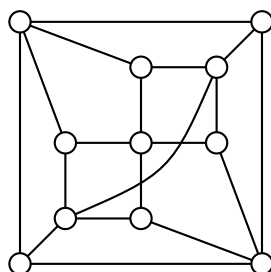
$a R b$ if, and only if $a + b$ is an even number.

(a) Prove that R is reflexive, symmetric and transitive. Describe its equivalence classes!

(b) Write down a description of R^C (in that specific case).

(c) Suppose R is defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ (with the same description). Draw its graph clearly.

3. [35 marks] You are given graph G in the figure below.



(a) Is the graph G Eulerian? If it is, mark its Eulerian tour, if it is not, explain why.

(b) Is the graph G Hamiltonian? If it is, draw its Hamiltonian cycle or use the graph disintegration theorem to show it is not.

(c) Determine the chromatic number of the graph G .