Matematika VSP: 3. računski izpit

17. avgust 2021

Čas pisanja je 60 minut. Dovoljena je uporaba 2 listov A4 formata s formulami. Uporaba elektronskih pripomočkov ni dovoljena.

Vse odgovore dobro utemelji!

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1. **[35 točk]** Dano je kompleksno število

$$a = 2 - 2i\sqrt{3}.$$

(a) Poišči njegov polarni zapis $a = re^{i\varphi}$. Jasno zapiši r in φ.

(b) Določi argument, tj. polarni kot števila a^{2021}.

(c) Poišči vse rešitve enačbe $z^2 = a$. Rešitve zapiši v obliki $z = x + yi$.

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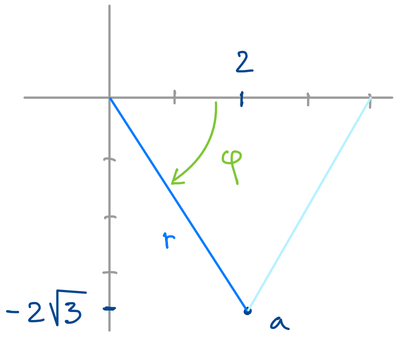
10 (a)



$r^2 = 2^2 + (-2\sqrt{3})^2 = 16 \; \dots \; r = 4$

$\varphi = -60° = -\frac{\pi}{3}$ (iz skice)

ali $\tan\varphi = \frac{-2\sqrt{3}}{2} = -\sqrt{3} \; \dots \; \varphi = -\frac{\pi}{3}$.

$a = 4e^{-i\pi/3}$

10 (b)

$a^{2021} = \left(4e^{-i\pi/3}\right)^{2021} = 4^{2021} e^{-i\frac{2021}{3}\pi} = 4^{2021} e^{i\pi/3}$.

$\frac{2021}{3}\pi = 674\pi - \frac{\pi}{3} = 337 \cdot 2\pi - \frac{\pi}{3}$

Argument a^{2021} je torej $\frac{\pi}{3}$.

15 (c)

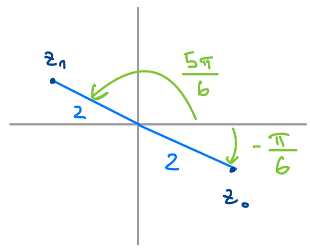
$z^2 = a \; \dots \; (re^{i\varphi})^2 = 4e^{-i\frac{\pi}{3}} \; \dots \; r^2 e^{2i\varphi} = 4e^{-i\frac{\pi}{3}} \; \dots$

$r^2 = 4 \; \dots \; r = 2$

$2\varphi_k = -\frac{\pi}{3} + 2k\pi \; \dots \; \varphi_k = -\frac{\pi}{6} + k\pi \; \dots$

$\varphi_0 = -\frac{\pi}{6} = -30°$

$\varphi_1 = \frac{5\pi}{6}$



$z_0 = 2e^{-i\pi/6} = \sqrt{3} - i$,

$z_1 = 2e^{i\frac{5\pi}{6}} = -\sqrt{3} + i$.

2. **[35 točk]** Dana je funkcija $f : [-1,1] \to \mathbb{R}$,

$$f(x) = (1 - x^2)(1 + x).$$

(a) Poišči njene ničle, lokalne ekstreme, intervale naraščanja in padanja, ter nariši njen graf.

(b) Določi največjo vrednost, ki jo zavzame funkcija f na intervalu $[-1,1]$.

(c) Izračunaj integral

$$\int_{-1}^{1} f(x)\,dx.$$

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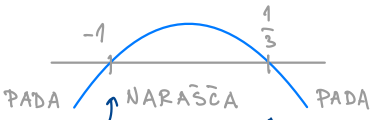
(a) Ničle: $(1-x^2)(1+x)=0 \;\ldots\; (1-x)(1+x)^2=0 \;\ldots\; x_1 = 1,\ x_{2,3} = -1$. $\underline{5}$

Stac. točke: $f'(x) = -2x(1+x) + (1-x^2)\cdot 1 = -3x^2 - 2x + 1 = -(3x-1)(1+x) = 0$

$\ldots\; x = \frac{1}{3},\ x = -1$, torej $\underline{5}$

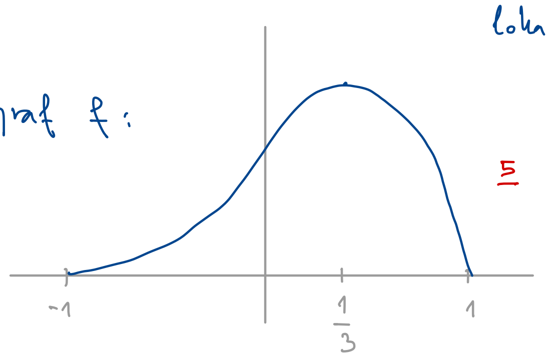


lokalni minimum

lokalni maksimum

$\underline{5}$

Graf f:



$\underline{5}$

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(b) Največjo vrednost f zavzame v stac. točki znotraj $[-1,1]$ ali na robu intervala:

$f(-1) = f(1) = 0$, $\quad f\left(\frac{1}{3}\right) = \frac{32}{27}$ ← to je največja vrednost.

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(c) $\displaystyle\int_{-1}^{1} (1-x^2)(1+x)\,dx = \int_{-1}^{1} (1 + x - x^2 - x^3)\,dx = \left(x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}\right)\Bigg|_{-1}^{1} =$

$= \frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{4}{3}.$

3. **[30 točk]** Dani sta matriki

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ -1 & 2 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} \text{ ter vektor } \vec{b} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}.$$

(a) Izračunaj produkt AB.

(b) Zapiši razširjeno matriko sistema $A\vec{x} = \vec{b}$, nato pa ta sistem reši z Gaussovo eliminacijo.

$10 \left\{ \text{(a)} \quad AB = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -3 & 1 \\ 1 & 3 \end{bmatrix}. \right.$

$20 \left\{ \text{(b)} \quad [A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -1 & 2 & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 1 & 2 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right] \ldots \ \vec{x} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}. \right.$