1.

$$A = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

Za PD poišči razcep Choleskega!

A: $|4| > 0$ ⋮ $|A| = 88 + 8 + 8 - (8 + 16 + 44) > 0$

$\begin{vmatrix} 4 & -2 \\ -2 & 2 \end{vmatrix} > 0$

$\Rightarrow$ A je PD!

B: $|1| > 0$

$\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} < 0 \Rightarrow$ B ni PD!

Cholesky

$A = A_1$

$a = 4, \; b^T = [-2, 2], \; B = \begin{bmatrix} 2 & -2 \\ -2 & 11 \end{bmatrix}$

$A_2 = B - \frac{1}{a} \cdot b b^T =$

$= \begin{bmatrix} 2 & -2 \\ -2 & 11 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} -2 \\ 2 \end{bmatrix} [-2, 2] =$

$= \begin{bmatrix} 2 & -2 \\ -2 & 11 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -1 & 10 \end{bmatrix}$

$a = 1, \; b = b^T = -1, \; B = [10]$

$A_3 = B - \frac{1}{a} b b^T =$

$= 10 - 1 = 9$

$a = 9$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 3 \end{bmatrix}$$

(2.) $A=\begin{bmatrix}1&0\\1&1\end{bmatrix}$; $\phi:\mathbb{R}^{2\times 2}\to\mathbb{R}^{2\times 2}$

$$\phi(X)=AXA^T+X$$

a) Preveri, da je ϕ linearna!

$$\phi(\alpha X+\beta Y)=A(\alpha X+\beta Y)A^T+(\alpha X+\beta Y)=$$
$$=(\alpha AX+\beta AY)A^T+\alpha X+\beta Y=$$
$$=\underline{\alpha AXA^T}+\beta AYA^T+\underline{\alpha X}+\beta Y$$
$$=\alpha(AXA^T+X)+\beta(AYA^T+Y)=$$
$$=\alpha\phi(X)+\beta\phi(Y)\ \checkmark$$

b) Poišči matriko lin preslikave!

$$\phi(E_{11})=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}1&0\\0&0\end{bmatrix}=$$
$$=\begin{bmatrix}1&0\\1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}1&0\\0&0\end{bmatrix}=$$
$$=\begin{bmatrix}1&1\\1&1\end{bmatrix}+\begin{bmatrix}1&0\\0&0\end{bmatrix}=\underline{\underline{\begin{bmatrix}2&1\\1&1\end{bmatrix}}}$$

$$\phi(E_{12})=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}0&1\\0&0\end{bmatrix}$$
$$=\begin{bmatrix}0&1\\0&1\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}0&1\\0&0\end{bmatrix}=$$
$$=\begin{bmatrix}0&1\\0&1\end{bmatrix}+\begin{bmatrix}0&1\\0&0\end{bmatrix}=\underline{\underline{\begin{bmatrix}0&2\\0&1\end{bmatrix}}}$$

$$\phi(E_{21})=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}0&0\\1&0\end{bmatrix}=$$
$$=\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}0&0\\1&0\end{bmatrix}=$$
$$=\begin{bmatrix}0&0\\1&1\end{bmatrix}+\begin{bmatrix}0&0\\1&0\end{bmatrix}=\underline{\begin{bmatrix}0&0\\2&1\end{bmatrix}}$$

$$\phi(E_{22})=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}0&0\\0&1\end{bmatrix}=$$
$$=\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}0&0\\0&1\end{bmatrix}=$$
$$=\begin{bmatrix}0&0\\0&1\end{bmatrix}+\begin{bmatrix}0&0\\0&1\end{bmatrix}=\underline{\underline{\begin{bmatrix}0&0\\0&2\end{bmatrix}}}$$

$$\begin{array}{cc} & \begin{array}{cccc}\phi(E_{11}) & \phi(E_{12}) & \phi(E_{21}) & \phi(E_{22})\end{array} \\ M= & \left[\begin{array}{cccc}2&0&0&0\\1&2&0&0\\1&0&2&0\\1&1&1&2\end{array}\right]\begin{array}{c}E_{11}\\E_{12}\\E_{21}\\E_{22}\end{array}\end{array}$$

c) Ali je 2 lastna vrednost ϕ?
Če je, kaj so pripadajoče lastne matrike?
Ali lahko ϕ diagonaliziramo?

$$\begin{vmatrix}2-\lambda&0&0&0\\1&2-\lambda&0&0\\1&0&2-\lambda&0\\1&1&1&2-\lambda\end{vmatrix}=(2-\lambda)^4=0$$

$\lambda_{1,2,3,4}=\underline{\underline{2}}$ => 2 $\underline{\text{je}}$ lastna vrednost ϕ!

$M-2I=$

$$\begin{bmatrix}0&0&0&0\\1&0&0&0\\1&0&0&0\\1&1&1&0\end{bmatrix}\sim$$

$$\sim\begin{bmatrix}\boxed{1}&0&0&0\\0&\boxed{1}&1&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$u=x_3\begin{bmatrix}0\\-1\\1\\0\end{bmatrix}\qquad v=x_4\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

Lastni matriki za lastn. vrednost 2:

$$\left\{\begin{bmatrix}0&-1\\1&0\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$$

ϕ ne moremo diag. Imamo 4-kratno lastno vrednost ($\lambda=2$) ampak samo 2 lin. neodvisni lastni matriki. Potrebovali bi 4.

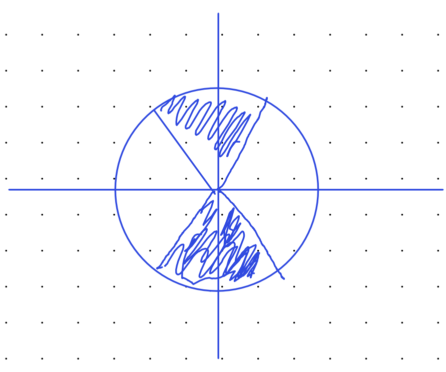
(3.) Telo T v $\mathbb{R}^3$ je dano z neenačbama

$$z^2 \geq x^2+y^2 \quad \text{in} \quad x^2+y^2+z^2 \leq 4$$

↳ stožec ... v obe smeri   krogla, $r=2$

Izračunaj:

$$\iiint_T z \, dx\,dy\,dz$$



Sferične koordinate!

$r \ldots [0, 2]$

$\varphi \ldots [0, 2\pi]$

$\theta \ldots [\frac{\pi}{4}, \frac{\pi}{2}]$

$$\iiint_T z \, dx\,dy\,dz =$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^2 r^3 \cdot \sin(\theta) \cdot \cos(\theta) \, dr \, d\varphi \, d\theta =$$

$$= 2\pi \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{4} r^4 \cdot \sin(\theta) \cdot \cos(\theta) \Big|_{r=0}^{2} d\theta =$$

$$= 8\pi \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(\theta) \cdot \cos(\theta) \, d\theta =$$

$t = \sin(\theta)$

$dt = \cos(\theta)$

$t \in [\frac{\sqrt{2}}{2}, 1]$

$$\int_{\frac{\sqrt{2}}{2}}^{1} t \, dt = \frac{1}{2} t^2 \Big|_{t=\frac{\sqrt{2}}{2}}^{1} =$$

$$= \frac{1}{2} \cdot \left(1 - \frac{2}{4}\right) = \frac{1}{4}$$

$$= 8\pi \cdot \frac{1}{4} = 2\pi$$

=> upošteval sem samo zgornjo polovico... končen rezultat je 2x toliko zaradi simetrije: 4π

ZA VAJO / SANITY CHECK...

recimo, da je $\rho(x,y,z) = 1$...

MASA:

$$m = \int_0^{2\pi} \int_0^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2 \cos(\theta) \, d\theta \, dr \, d\varphi =$$

$$= 2\pi \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{3} r^3 \cos(\theta) \Big|_{r=0}^{2} d\theta$$

$$= \frac{16\pi}{3} \cdot \sin\theta \Big|_{\theta=\frac{\pi}{4}}^{\frac{\pi}{2}}$$

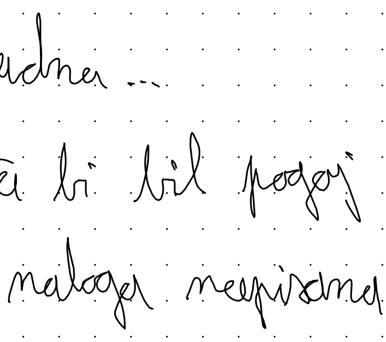
$$= \frac{16\pi}{3} \cdot \left(1 - \frac{\sqrt{2}}{2}\right)$$

MASNO SREDIŠČE (z^*)

$$z^* = \frac{1}{m} \iiint_T z \, dx\,dy\,dz =$$

$$= \frac{3}{8\cancel{16\pi} \cdot \left(1 - \frac{\sqrt{2}}{2}\right)} \cdot \cancel{2\pi} =$$

$$\doteq 1{,}28$$



Ta naloga je čudna...

Razumel bi, če bi bil pogoj $z \geq 0$.

Tako kot je pa naloga napisana, bi pa integral moral pridit $\underline{\underline{0}}$.

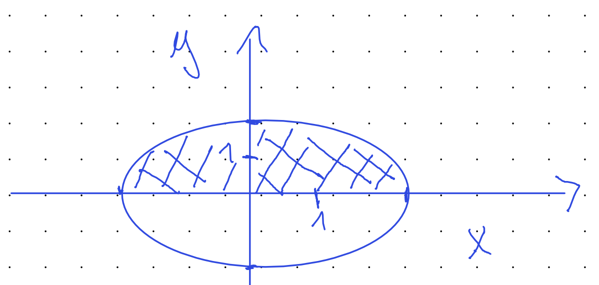
4.) Poišči min in max funkcije

$$f(x,y) = x^2 + (y-1)^2$$

pri pogojih

$$x^2 + 2y^2 \leq 4 \text{ in } y \geq 0$$

$$\frac{x^2}{4} + \frac{y^2}{2} \leq 1 \wedge y \geq 0$$



NOTRANJOST / GLOBAL. EKSTREM:

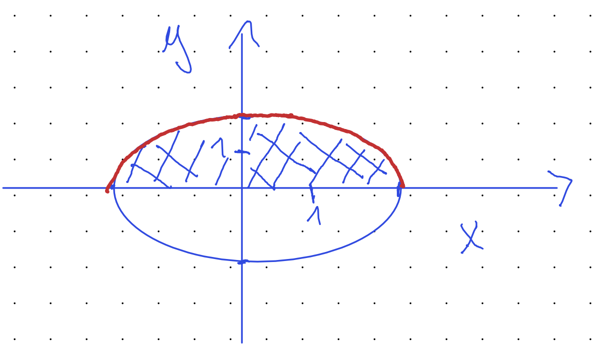
$$f_x = 2x = 0$$

$$f_y = 2(y-1)\cdot 1 =$$
$$= 2y - 2 = 0$$

$$\left.\begin{array}{l} x = 0 \\ y = 1 \end{array}\right\} K_1 = (0,1)$$

ROB (ZGORNJI)

$$x^2 + 2y^2 = 4, \quad y > 0$$



$$L(x,y,\lambda) = x^2 + (y-1)^2 - \lambda(x^2 + 2y^2 - 4)$$

$$\frac{\partial L}{\partial x} = 2x - 2\lambda x = 0$$

$$\frac{\partial L}{\partial y} = 2(y-1)\cdot 1 - 4\lambda y = 0$$

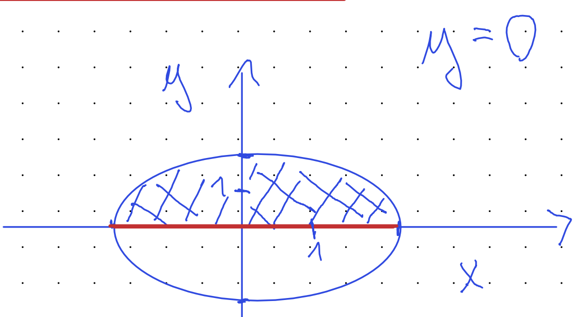
$$\frac{\partial L}{\partial \lambda} = -(x^2 + 2y^2 - 4) = 0$$

- $x(1-\lambda) = 0$
  - (1) $x = 0$ ... $K_2(0, \sqrt{2})$
  - (2) $\lambda = 1$ //

- $2y - 2 = 4\lambda y$
  $$y - 1 = 2\lambda y$$
  $$y(1 - 2\lambda) = 1$$
  $$y = \frac{1}{1-2\lambda}$$
  $$\lambda < \frac{1}{2}$$

ROB (SPODNJI)

$y = 0$



$$L(x,y,\lambda) = x^2 + (y-1)^2 - \lambda y$$

$$\frac{\partial L}{\partial x} = 2x = 0$$

$$\frac{\partial L}{\partial y} = 2(y-1) - \lambda = 0$$

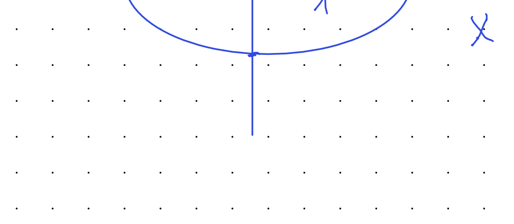
$$\frac{\partial L}{\partial \lambda} = -y = 0$$

$$x = 0$$
$$2y - 2 - \lambda = 0$$
$$y = 0 \qquad K_3(0,0)$$

PRESEK ROBOV

$K_4(2,0)$

$K_5(-2,0)$



$$f(K_1) = f(0,1) = 0$$
$$f(K_2) = f(0,\sqrt{2}) = (\sqrt{2}-1)^2$$
$$f(K_3) = f(0,0) = 1$$
$$f(K_4) = f(2,0) = 4$$
$$f(K_5) = f(-2,0) = 4$$

K_1 ... min $f(K_1) = 0$

K_4, K_5 ... max $f(K_4) = f(K_5) = 4$