① a) $K(x,y) = 2x^2 + 4xy + 5y^2$ 5

b) $\det\begin{bmatrix} 2-\lambda & 2 \\ 2 & 5-\lambda \end{bmatrix} = (2-\lambda)(5-\lambda) - 4 = 6 - 7\lambda + \lambda^2 = (\lambda-1)(\lambda-6)$ 5

$\lambda_1 = 1,\ \lambda_2 = 6$

kan. oblika $u^2 + 6v^2$

c) $\lambda = 1$: $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $v_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ 5

$\lambda = 6$ $\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$ $x = \frac{1}{\sqrt{5}}(2u + v)$, $y = \frac{1}{\sqrt{5}}(-u + 2v)$

d) Ker je $\det A = 10 - 4 = 6 > 0$ in $2 > 0$, ima f v (a,b) lok. minimum.

5

② a) D_f: $x^2 - y > 0$ 5

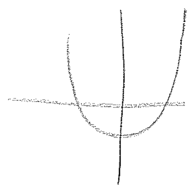
$y < x^2$

b) $f(1,0) = \log 1 = 0$

nivojnica $\log(x^2 - y) = 0$

$x^2 - y = 1$

$\boxed{y = x^2 - 1}$



5

c) $\operatorname{grad} f = \left(\frac{2x}{x^2 - y}, \frac{-1}{x^2 - y} \right)$ 5

d) smer y-osi: $(0,y)$ $\frac{2x}{x^2 - y} = 0$

$x = 0$. v vsaki točki $(0,y)$ 5

e) $f_{\left[\begin{smallmatrix}2\\1\end{smallmatrix}\right]}(1,0) = \frac{1}{\sqrt{5}} \left(\frac{2}{1}, \frac{-1}{1} \right) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{5}} \cdot 3 = \frac{3}{\sqrt{5}} > 0 \Rightarrow$ povečala. 5

④ a) $\frac{dy}{y} = \frac{2}{x} dx$ 10

$\log y = 2\log x + C$

$y = e^{\log x^2 + C} = Dx^2$

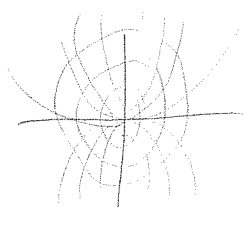
b) $1 = D \cdot 1 \Rightarrow D = 1$ 5

c) $-x \frac{1}{y'} = 2y$

$-x\,dx = 2y\,dy$ 10

$-\frac{x^2}{2} = 2\frac{y^2}{2} + C$

$y^2 = -\frac{x^2}{2} - C$



$\boxed{3.}$ $g(x,y) = x^2 - x^2y^2 + y^2$

(a) 7

$$\frac{\partial g}{\partial x} = 2x - 2xy^2 = 2x(1-y^2) = 0$$

2

$$\frac{\partial g}{\partial y} = 2y - 2x^2y = 2y(1-x^2) = 0$$

$y = 0$ → $2x = 0$ ⇓ $x = 0$

$(x,y) = (0,0)$

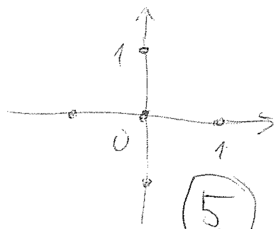
$T_1(0,0)$

→ $1 - x^2 = 0$ oz $x = \pm 1$

$\Rightarrow 1 - y^2 = 0$

$y = \pm 1$

$T_{2,3,4,5}(\pm 1, \pm 1)$



5

(b) 6

$$H_g = \begin{bmatrix} \frac{\partial^2 g}{\partial x^2} & \frac{\partial^2 g}{\partial x \partial y} \\ \frac{\partial^2 g}{\partial x \partial y} & \frac{\partial^2 g}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 2-2y^2 & -4xy \\ -4xy & 2-2x^2 \end{bmatrix}$$

2

$H_g(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $\det H_g(0,0) = 4 > 0$, $2 > 0$

$\Rightarrow$ $T_1(0,0)$ je minimum.

2

$H_g(\pm 1, \pm 1) = \begin{bmatrix} 0 & \pm 4 \\ \pm 4 & 0 \end{bmatrix}$, $\det H_g(\pm 1, \pm 1) = -16 < 0$ ni ekstrema

2

(c) $L(x,y,\lambda) = g(x,y) - \lambda\,(x^2 + y^2 - 1) =$

12

$= x^2 - x^2 y^2 + y^2 - \lambda\,(x^2+y^2-1)$

$\frac{\partial L}{\partial x} = 2x - 2xy^2 - 2x\lambda = 0 \quad \therefore \quad 2x(1-y^2-\lambda) = 0$

5

$\frac{\partial L}{\partial y} = 2y - 2x^2y - 2y\lambda = 0 \quad \therefore \quad 2y\,(1-x^2-\lambda) = 0$

$\frac{\partial L}{\partial \lambda} = -(x^2+y^2-1) = 0$

$y = 0$
$x = \pm 1$

ali

$1 - x^2 - \lambda = 0$
$\lambda = 1 - x^2$
$x\,(-y^2 + x^2) = 0$

$S_{1,2}(\pm 1, 0)$

$x = 0$
$y = \pm 1$
$S_{3,4}(0, \pm 1)$

$x^2 = y^2$
$2x^2 = 1$
$x^2 = \frac{1}{2}$
$x = \pm\sqrt{\frac{1}{2}} = y$
$S_{5,6,7,8}\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$

5

$f(S_{1,2}) = 1$
$f(S_{3,4}) = 1$ 1
$f(S_{5-8}) = \frac{1}{2} - \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ $\frac{3}{4}$

8 točk, če S_{1-4}. 2

5

(d) $f(T_1) = 0$

5

~~$f(\ldots) = 1$~~ 0, 1